Linear-Phase FIR Filter Segment Synthe

Based on an 1995 IEEE paper co-authorded by Amin G Jaffer & William E Jones

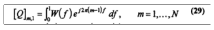
1 Global Definitions

```
(%i1)  assume(A>=0,B>=0);
(%o1)  [A>=0,B>=0]
```

```
(%i2)  declare(p,integer,q,integer);
(%o2)  done
```

2 General result

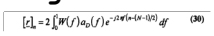
Figure 1:



```
(%i3)  Q[L]:'integrate(W(f)*exp(%i*2*%pi*f*L),f);
```

$$(\%o3)\quad \int \%e^{2\,\%i\,\pi\,L\,f}\,W(f)\,df$$

Figure 2:



```
(%i4)  R[p]:'integrate(W(f)*a(f)*exp(-%i*2*%pi*f*(p-%Lambda)),f);
```

$$(\%o4)\quad \int W(f)\,a(f)\,\%e^{-2\,\%i\,\pi\,f\,(p-\Lambda)}\,df$$

3 Normal Error

3.1 Normal Error Exponential Segment

```
(%i5)  Aexp:exp(A+f*B);
(Aexp) %e^(B f+A)
```

Q-Segment

(%i6)
```
Qen0[L]:(ev(Q[L],W(f):1));
```
(%o6)
$$\int \%e^{2\,\%i\,\pi\,L\,f}\,\mathrm{d}f$$

Q

(%i7)
```
Qen[L]:ev(Qen0[L],integrate);
```
(%o7)
$$-\frac{\%i\,\%e^{2\,\%i\,\pi\,L\,f}}{2\,\pi\,L}$$

Q, Denom==0 [L==0]

(%i8)
```
QenL[p,q]:ev(Qen0[L],L:0,integrate);
```
(%o8)
$$f$$

R-Segment

(%i9)
```
Ren0[p]:(ev(R[p],W(f):1,a(f):Aexp));
```
(%o9)
$$\int \%e^{-2\,\%i\,\pi\,f\left(p-\Lambda\right)+B\,f+A}\,\mathrm{d}f$$

R

(%i10)
```
Ren[p]:ev(Ren0[p],W(f):1,integrate);
```
(%o10)
$$\frac{\%e^{-2\,\%i\,\pi\,f\left(p-\Lambda\right)+B\,f+A}}{B-2\,\%i\,\pi\left(p-\Lambda\right)}$$

R, Denom==0 [(p==Lambda) && (B==0)]

(%i11)
```
RenD[p]:ev(Ren0[p],B:0,p:%Lambda,integrate);
```
(%o11)
$$\%e^{A}\,f$$

3.2 Normal Error Linear Segment

(%i12)
```
Alin:A+f*B;
```
(Alin)
$$B\,f+A$$

Q-Segment

```
(%i13)  Qln0[L]:ev(Q[L],W(f):1);
```

$$(\%o13)\quad \int \%e^{2\,\%i\,\pi\,L\,f}\,\mathrm{d}f$$

Q

```
(%i14)  Qln[L]:ev(Qln0[L],integrate);
```

$$(\%o14)\quad -\frac{\%i\,\%e^{2\,\%i\,\pi\,L\,f}}{2\,\pi\,L}$$

Q, Denom==0 [L==0]

```
(%i15)  QlnD[L]:ev(Qln0[L],L:0,integrate);
(%o15)  f
```

R-Segment

```
(%i16)  Rln0[p]:ev(R[p],a(f):Alin,W(f):1);
```

$$(\%o16)\quad \int \left(B\,f+A\right)\%e^{-2\,\%i\,\pi\,f\left(p-\Lambda\right)}\,\mathrm{d}f$$

R

```
(%i17)  Rln[p]:factor(radcan(ev((Rln0[p]),integrate)));
```

$$(\%o17)\quad \frac{\left(2\,\%i\,\pi\,B\,f\,p+2\,\%i\,\pi\,A\,p-2\,\%i\,\pi\,\Lambda\,B\,f+B-2\,\%i\,\pi\,\Lambda\,A\right)\%e^{2\,\%i\,\pi\,\Lambda\,f-2\,\%i\,\pi\,f\,p}}{4\,\pi^2\left(p-\Lambda\right)^2}$$

R,Denom(R0)==0 [p==Lambda]

```
(%i18)  RlnD[p]:ev(Rln0[p],p:%Lambda,integrate);
```

$$(\%o18)\quad \frac{B\,f^2}{2}+A\,f$$

4 Relative Square-Error

4.1 Exponential Segment Relative-Square Error

```
(%i19)  Aexp:exp(A+f*B);
(Aexp)  %e^(B f+A)
```

Q-Segment

(%i20)
```
Qer0[L]:(ev(Q[L],W(f):Aexp^-2));
```

(%o20)
$$\int \%e^{2\,\%i\,\pi\,L\,f-2\,\left(B\,f+A\right)}\,\mathrm{d}f$$

Q

(%i21)
```
Qer[L]:ev(Qer0[L],integrate);
```

(%o21)
$$\frac{\%e^{2\,\%i\,\pi\,L\,f-2\,\left(B\,f+A\right)}}{2\,\%i\,\pi\,L-2\,B}$$

Q, Denom==0 [(L==0)&&(B==0)]

(%i22)
```
QerD[L]:ev(Qer0[L],L:0,B:0,integrate);
```

(%o22)
$$\%e^{-2\,A}\,f$$

R-Segment

(%i23)
```
Rer0[p]:((ev(R[p],W(f):1,a(f):Aexp^-1)));
```

(%o23)
$$\int \%e^{-2\,\%i\,\pi\,f\,\left(p-\Lambda\right)-B\,f-A}\,\mathrm{d}f$$

R

(%i24)
```
Rer[p]:ev(Rer0[p],integrate);
```

(%o24)
$$\frac{\%e^{-2\,\%i\,\pi\,f\,\left(p-\Lambda\right)-B\,f-A}}{-2\,\%i\,\pi\,\left(p-\Lambda\right)-B}$$

R, Denom==0 [(p==Lambda)&&(B==0)]

(%i25)
```
RerD[p]:ev(Rer0[p],p:%Lambda,B:0,integrate);
```

(%o25)
$$\%e^{-A}\,f$$

4.2 Relative-Square Error Linear Segment

(%i26)
```
Alin:A+f*B;
```

(Alin)
$$B\,f+A$$

Q-Segment

```
(%i27)  Qlr0[L]:(ev(Q[L],W(f):Alin^-2));
```

$$(\%o27)\quad \int \frac{\%e^{2\,\%i\,\pi\,L\,f}}{\left(B\,f+A\right)^2}\,\mathrm{d}f$$

Q

```
(%i28)  Qlr[L]:ev(Qlr0[L],integrate);
```

$$(\%o28)\quad -\frac{\mathrm{expintegral\_e}\left(2,-\frac{2\,\%i\,\pi\,L\left(B\,f+A\right)}{B}\right)\,\%e^{-\frac{2\,\%i\,\pi\,A\,L}{B}}}{B\left(B\,f+A\right)}$$

Q, B==0

```
(%i29)  QlrB[L]:ev(Qlr0[L],B:0,integrate);
```

$$(\%o29)\quad -\frac{\%i\,\%e^{2\,\%i\,\pi\,L\,f}}{2\,\pi\,A^2\,L}$$

Q, L==0

```
(%i30)  QlrL[L]:ev(Qlr0[L],L:0,integrate);
```

$$(\%o30)\quad -\frac{1}{B\left(B\,f+A\right)}$$

Q, B==0 && L==0

```
(%i31)  QlrBL[L]:ev(Qlr0[L],L:0,B:0,integrate);
```

$$(\%o31)\quad \frac{f}{A^2}$$

R-Segment

```
(%i32)  Rlr0[p]:(ev(R[p],W(f):1,a(f):Alin^-1));
```

$$(\%o32)\quad \int \frac{\%e^{-2\,\%i\,\pi\,f\left(p-\Lambda\right)}}{B\,f+A}\,\mathrm{d}f$$

R

(%i33)
```
Rlr[p]:ev(Rlr0[p],integrate);
```

(%o33)
$$-\frac{\text{expintegral\_e}\left(1,\frac{2\,\%i\,\pi\,\left(B\,f+A\right)\left(p-\Lambda\right)}{B}\right)\,\%e^{\frac{2\,\%i\,\pi\,A\,\left(p-\Lambda\right)}{B}}}{B}$$

R, Denom==0 [B==0]

(%i34)
```
RlrB[p]:ev(Rlr0[p],B:0,integrate);
```

(%o34)
$$\frac{\%i\,\%e^{-2\,\%i\,\pi\,f\,\left(p-\Lambda\right)}}{2\,\pi\,A\,\left(p-\Lambda\right)}$$

R, p==Lambda

(%i35)
```
RlrL[p]:ev(Rlr0[p],p:%Lambda,integrate);
```

(%o35)
$$\frac{\log\left(B\,f+A\right)}{B}$$

R, B==0, p==Lambda

(%i36)
```
RlrBL[p]:ev(Rlr0[p],p:%Lambda,B:0,integrate);
```

(%o36)
$$\frac{f}{A}$$